**Homework 4**

1. Consider steady, undeveloped laminar flow through a cylindrical blood vessel with a diameter of 1 cm, with an entrance length of 1 cm. Assume the velocity of blood in the vessel is 10 cm/s and the viscosity of the blood is 0.03 cm2/s. Assume a solute within the blood has a diffusivity of 10-6 cm2/s. The concentration of the solute in the blood is 5 mM; assume the surface concentration is 0. Determine **(A)** the Reynolds and Schmidt numbers, **(B)** the Sherwood number and the mass transfer coefficient, and **(C)** the total mass transfer rate. ***(Ans: )***

**Ans:**

**Part A**

Reynolds number for a cylindrical tube,

Schmidt number,

**Part B**

Test for flow development:

So, the flow is undeveloped.

Test for concentration boundary development:

So, the concentration boundary is undeveloped.

Sherwood number,

Mass transfer coefficient,

**Part C**

Mass transfer rate,

1. Consider a spherically shaped molecule that has a molecular weight of 42 kDa through a microporous polycarbonate membrane. Assume that the molecule enters the pores of the membrane from a well-stirred bulk solution (i.e., there is polarization region) that is at 37oC (assume solvent is water). The membrane pores are cylindrical and have a diameter of 32 nm with an overall length of 325 mm. The membrane itself is 250 mm thick. The pores comprise 42% of the membrane volume (hint: this should not be confused with the porosity; you will need to find that). Determine **(A)** the solute molecule radius and its bulk diffusion coefficient, **(B)** the partition coefficient accounting for both steric exclusion and hindered diffusion (i.e., ), **(C)** the porosity and tortuosity of the membrane, and **(D)** the permeability of the membrane for the solute.

**(Ans: )**

**Part A**

For a spherical molecule,

Diffusion coefficient using Renkin and Curry,

**Part B**

Partition coefficient for steric exclusion and hindered diffusion,

**Part C**

Tortuosity,

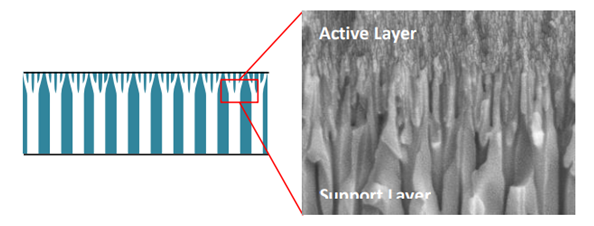
Pore volume,

**Part D**

Permeability,

1. Consider the transport of glucose (C6H12O6) across an anodic aluminum oxide (AAO) membrane as seen in the figure. The membrane has a diameter of 13 mm and an overall thickness of 50 mm. The membrane is composed of two layers: an active layer with very small pores and a support layer with larger pores. The active layer, which makes up 5% of the membrane thickness, has an average pore size of 6 nm in diameter with a porosity of 12%. The support layer (making up the remaining 95% of the membrane thickness) has pores that are 200 nm diameter in size with a porosity of 18%. Assume the pores are straight. The membrane separates two well-stirred volumes. One side has an average glucose concentration of 20 mM. The other side contains no glucose, but 5mM of a large macromolecule that is impermeable to the membrane. Assume the solvent is water at 37oC. Assume the hydrodynamic pressure of both volumes on either side of the membrane are constant at 1 atm. Determine the following: **(A)** the overall hydraulic conductance of the membrane, **(B)** the filtrate flow rate through the membrane, **(C)** the actual sieving coefficient (hint: for a two-layer membrane like this, it will the product of the two individual sieving coefficients), **(D)** the overall permeability of the membrane to glucose, **(E)** the total glucose transfer rate across the membrane, and **(F)** the Peclet number for the membrane.

**(Ans: ; )**



**Ans:**

**Part A**

Hydraulic conductance of membranes,

Conductance in series will combine like resistance in parallel:

**Part B**

Both sides have the same hydrodynamic pressure, right side has impermeable macromolecule that generates osmotic pressure difference.

Filtrate flow rate through the membrane:

**Part C**

Radius of glucose,

For support layer,

For active layer,

Sieving coefficient,

**Part D**

Diffusion coefficient using Renkin and Curry,

Permeability,

**Part E**

Overall solute transport,

**Part F**

Peclet number,

1. A technique known as fluorescence recovery after photobleaching (FRAP) is used to measure the diffusivity of fluorescein (MW=332 Da), a fluorescent tracer, in the porcine temporomandibular joint (TMJ) disk. The TMJ disk consists mostly of water and large a large amount of Type I collagen. The overall average diffusivity of fluorescein in the TMJ disk was found to be 57.0 m2/s. Use the Brinkman model to estimate the value of for the fluorescein diffusion in the porcine TMJ disk. Assume that the measurements were made at . **(Ans: )**

Ans:

Solute radius,

Diffusivity,

Brinkman model,

Solving for k gives weird value of 347514139

1. Determine **(A)** the maximum inter-capillary half distance (or critical radius) for skeletal muscle given that the diffusion coefficient of oxygen in the interstitial tissue is and the reaction rate of oxygen in the skeletal muscle tissue is . Assume the oxygen concentration at the capillary surface to be constant at . Assume the capillaries in the skeletal muscle have (external) diameters of 3 mm. **(B)** Repeat the same calculations for a capillary in the brain where the reaction rate is and the capillary diameter is 5 mm; all other parameters are the same. (Hint: the maximum critical radius occurs at the beginning when . Also, since you were given the surface concentration, you do not need to include the term that accounts for transport across the capillary wall.) **(Ans: )**

**Part A**

Concentration of oxygen using Krogh Cylinder model,

Maximum critical radius at

Surface concentration given.

Thus,

Using solver, (doesn’t match given answer).

Part B

For

Using solver,